THE CHINESE UNIVERSITY OF HONG KONG
DEPARTMENT OF MATHEMATICS

MMAT5210 Discrete Mathematics 2017-2018
Assignment 1 (Due date: 1 Feb, 2017)

1. Define a relation $\sim$ on $\mathbb{R}^2$ such that $(x_1, y_1) \sim (x_2, y_2)$ if and only if $x_1^2 + y_1^2 = x_2^2 + y_2^2$.
   (a) Show that the relation $\sim$ is an equivalence relation.
   (b) What are the elements of the equivalence class $[(3, 4)]$?
   (c) Describe the elements of $\mathbb{R}^2/\sim$.
2. Let $\mathbb{R}[x]$ be the set of all polynomials with real coefficients.
   Define a relation $\sim$ on $\mathbb{R}[x]$ such that $P(x) \sim Q(x)$ if and only if $P(x) - Q(x)$ is divisible by $x^2 + x + 1$.
   (a) Show that the relation $\sim$ is an equivalence relation.
   (b) Describe the elements of $\mathbb{R}[x]/\sim$.
3. Construct a concrete bijective function from $\mathbb{Z}^+$ to $\mathbb{Z}$.
4. Show that $\log(n^2 + 1)$ is $O(\log n)$.
5. (a) Show that $n \log n$ is $O(\log n!)$.
   (b) Is it true that $n \log n$ is $\Theta(\log n!)$?
6. Let $\alpha > \beta > 1$. Show that β^n is $O(\alpha^n)$, but α^n is not $O(\beta^n)$.
7. Let k be a positive integer.
   (a) Show that $1^k + 2^k + \cdots + n^k$ is $O(n^{k+1})$.
   (b) Is it true that $1^k + 2^k + \cdots + n^k$ is $\Theta(n^{k+1})$?
8. (a) Show that for any positive integer $n \geq 2$,
   $$\sum_{j=2}^{n} \frac{1}{j} < \int_1^n \frac{1}{x}\, dx.$$
   (b) Let H_n be the n-th harmonic number
   $$H_n = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}.$$
   By using the result in (a), show that H_n is $O(\log(n))$